Boundary Layer Equations for Two Parameter Group Theoretic method of Power Law fluid Over a Porous Medium

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Abstract

In the present paper, we have investigated the problem to observe the behaviour of stagnation point flow of power law fluid over a porous flat plate. The governing non-linear partial differential equations are reduced to an ordinary differential equation using a deductive group theoretic method, which reduces the number of independent variables to one. Similarity solutions for all possible cases are obtained for non-Newtonian boundary layer flow. Further, a Numerical solution is obtained using bvp4c in MATLAB. The influence of different flow parameters on the velocity profile is analysed and presented graphically.

Keywords: Deductive group-theoretic method, Power law fluid, Similarity solutions, Porous Medium.

Nomenclature

G	Group
a_1, a_2	Group parameter
a_1^0, a_2^0	Identity element of a Group
n	Fluid index
F	Arbitrary function
u_e	velocities in x directions
u, v	Velocity component in the X-direction and Y-direction
x, y, t	rectangular coordinates
τ	Stress component
ψ	Stream function
η	Independent similarity variable
p	Pressure
ρ	Density
B_0	Uniform transverse magnetic field
σ_e	The electrical conductivity
Re	Reynolds number
A, C, K	Constants of fluid models
λ	Positive rheological constant
$\square^{S}, \square^{S}$	real-valued and differentiable in their real argument '(a_1, a_2)'.

1. Introduction

Fluid mechanics gives the concept of fluids at rest or in motion. There are many real, practical applications of fluid dynamics that motivate the study of fluids and their behaviour in different situations. To study Fluids, different techniques and methods have been developed in the past many years, and it can be useful to understand the nature of fluids. Since many fluids have a non-Newtonian nature and due to their wide application in the industrial field, research in the field of non-Newtonian fluids has increased with an increase in demand.

Observation of such literature surveys is that Helge I. Andersson et al. (Andersson & Toften, 1989) worked on the numerical solution for the laminar boundary layer problem for power law fluid by using the resemblance between the turbulent eddy viscosity and the effective viscosity, for non-Newtonian behaviour. M. G. Timol et al. (Darji & Timol, 2013) worked on general group symmetry analysis by deductive group-theoretical method to analyze boundary layer flow of electrically conducting viscous fluid with heat transfer over a non-linear porous surface. R. M. Darji et al. (Darji & Timol, 2014) worked on the deductive group symmetry approach to the system of unsteady natural convection boundary layer flow of the non-Newtonian over a non-isothermal vertical flat plate for two parameters. S. Dholey (Dholey, 2018) studied different solutions by a group theoretic approach applied to analyze the unsteady separated stagnation-point flow over a moving flat plate immersed in a non-Newtonian power-law fluid. A study of the rheological behaviour of the fluid for unsteady stagnation point flows of power law fluid over a porous flat plate with mass transfer is analysed for two numerical solutions, AFS and RFS, by S. Dholey (Dholey, 2020). Nisiki Hayasi (Hayasi, 1965) explained the conditions for the existence of similar solutions for the two-dimensional and axisymmetric boundary layers obtained for the steady or unsteady flow for purely viscous non-Newtonian fluids. A study of Boundary layer flow for a power law non-Newtonian fluid in the presence of a transverse magnetic field applied perpendicular to the surface is explained by F. N. Ibrahim et al. (Ibrahim & Terbeche, 1994).

Nita Jain et al. (Jain & Timol, 2015) employed deductive group theoretic transformation to develop a similarity solution of steady laminar incompressible quasi three-dimensional boundary layer flow for power law fluid for all possible cases. Tsung Yen Na et al. (Na & Hansen, 1967) in their work analysed the similarity solutions for the three-dimensional steady incompressible boundary layer equation for power law fluid in rectangular co-ordinates. S. Nadeem et al. (Nadeem et al., 2013) studied magnetohydrodynamic (MHD) Casson fluid flow in two lateral directions past a porous linear stretching sheet and obtained self-similar solutions, and compared with available data for special cases. Analytical solutions of steady laminar boundary layer non-Newtonian flow with non-linear viscosity over a moving flat plate for power law fluid are investigated by Jacob Nagler (Nagler, 2014). M. G. Timol et al. (Parmar & Timol, 2011) worked on the effects of boundary layer equations for coupled heat and mass transfer, natural convection of a viscous incompressible and electrically conducting flow of non-Newtonian power law fluid over a vertical permeable cone surface through porous medium in the presence of a uniform transverse magnetic field and thermal radiation. Hiral Parmar et al. (Parmar & Timol, 2012) analyzed the behaviour of two-dimensional unsteady boundary layer flow of a micropolar non-Newtonian power law fluid near the stagnation point in a porous medium. Jayshri Patel et al. (Patel et al., 2015) worked on steady laminar two-dimensional boundary layer flow of non-Newtonian fluids for the Sisko fluid model.

Numerical solution is derived using FDM on a moving surface in MDH boundary layer flow of non-Newtonian Sisko fluid for third-order ODE by Jayshri Patel et al. (Patel et al., 2019). Manisha Patel et al. (Patel et al., 2021) analyzed the extension of the Blasius Newtonian Boundary layer to Blasius non-Newtonian boundary layer using some fluid models like Power law model, Sisko model, and Prandtl model. A study of similarity solutions over a continuously moving surface in the presence of a transverse magnetic field for a non-Newtonian power law fluid is given by Govind R. Rajput et al. (Rajput et al., 2014). A study of unsteady boundary layer stagnation point flows and heat transfer over a stretching sheet by considering porosity is given by Haliza Rosali et al. (Rosali et al., 2020). Hemangini S.

Shukla et al. (H. S. Shukla et al., 2020) studied MHD nanofluid flow over a linearly stretching sheet in three dimensions for a non-Newtonian power law model with convective boundary conditions. The effect of nanoparticles volume fraction on the temperature and velocity profile is analyzed using a similarity solution on a three-dimensional nanofluid over a flat surface stretched continuously in two lateral directions by Hemangini Shukla et al. (H. Shukla et al., 2018). Hemangini Shukla et al. (H. Shukla et al., 2022) worked on unsteady Sisko flow with viscous dissipation using the similarity technique and the effects of heat transfer in fluid flow.

Atta Sojoudi et al. (Sojoudi et al., 2014) obtained the similarity solutions of a non-Newtonian power law fluid and heat transfer for an unsteady stretching surface. Hema C. Surati et al. (Surati et al., 2016) worked on generalized analysis to develop a similarity solution of steady two dimensional boundary layer flow of non-Newtonian Sisko fluid, where the boundary value problem is transformed into an initial value problem. M. G. Timol et al. (Timol & Kalthia, 1986) explained that for a non-Newtonian fluid of any model, a similarity solution exists for the fluid where shearing stress and rate of strain can be related by an arbitrary continuous function. Md. Jashim Uddin (Uddin, 2015) worked on unsteady hydromagnetic boundary layer flow of an incompressible and electrically conducting fluid through a porous medium for a moving surface. Two-dimensional unsteady boundary layer flow past a shrinking sheet with suction effect at the surface for a non-Newtonian power law fluid was investigated by Nor Azizah Yacob et al. (Yacob et al., 2012).

In this paper, similarity solutions are obtained by applying the two-parameter deductive group theoretic method to the governing partial differential equations, thereby reducing them to a system of an ordinary differential equation. The reduced ordinary differential equation is solved numerically using bvp4c solver in MATLAB. The effects of various flow parameters and fluid index on the velocity profile are studied and presented graphically.

2. Governing Equations

Consider the unsteady two-dimensional power-law fluids for stagnation-point boundary layer flows over a permeable surface coinciding with the plane y = 0 in the presence of uniform transverse magnetic field B_0. We assume the outer flow velocity as $u_e(x,t)$. The governing equations of continuity and momentum for unsteady flow problem in dimensionless form (Dholey, 2020) are given as:

Continuity equation:

$$\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}+\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)-\frac{\sigma_e B_0^2}{\rho}u \tag{2}$$

where external flow velocity $u_e(x,t)$ relates to the pressure p by the following relation:

$$\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}=-\frac{\partial p}{\partial x} \tag{3}$$

Together with the boundary conditions

$$\begin{aligned} y&=0, \qquad u(x,0,t)=0=v(x,0,t) \\ y&=\infty, \qquad u(x,y,t)=u_e(x,t) \end{aligned} \tag{4}$$

3. Mathematical formulation

Introducing the stream function $\psi(x,y,t)$ such that

$$u=\frac{\partial\psi}{\partial y} \quad \text{and} \quad v=-\frac{\partial\psi}{\partial x} \tag{5}$$

Substitute the above values in (1), (2) and (4) we can say that continuity equation will be satisfied automatically and momentum equation is written as:

$$\frac{\partial^2\psi}{\partial t\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}+\frac{\partial}{\partial y}\left(\left|\frac{\partial^2\psi}{\partial y^2}\right|^{n-1}\frac{\partial^2\psi}{\partial y^2}\right)-\frac{\sigma_e B_0^2}{\rho}\frac{\partial\psi}{\partial y} \tag{6}$$

with the boundary conditions:

$$\begin{aligned} y&=0, \qquad \frac{\partial\psi}{\partial y}(x,0,t)=0=\frac{\partial\psi}{\partial x}(x,0,t) \\ y&=\infty, \qquad \frac{\partial\psi}{\partial y}(x,y,t)=u_e(x,t) \end{aligned} \tag{7}$$

3.1 Deductive Group Theoretic method

Here, on equation (6) and boundary conditions (7) we will apply two-parameter deductive group theoretic technique. By using this method three independent variables will be reduced to one, and partial differential equations will be reduced to an ordinary differential equation.

We introduce two parameter group of transformation as,

$$G: \quad \overline{S}=\square^{S}(a_1,a_2)S+\square^{S}(a_1,a_2), \qquad S=x,y,t,\psi,u_e \tag{8}$$

where $'(a_1,a_2)'$ are the two parameters of the transformation. $\square\,'s$ and $\square\,'s$ are real-valued and at least differential in each real argument (a_1,a_2).

Equation (6) remain invariant under group of transformations defined by G in equation (8), for some function $\mathscr{H}_1(a_1,a_2)$, whenever

$$\begin{aligned} &\frac{\partial^2\overline{\psi}}{\partial\overline{t}\partial\overline{y}}+\frac{\partial\overline{\psi}}{\partial\overline{y}}\frac{\partial^2\overline{\psi}}{\partial\overline{x}\partial\overline{y}}-\frac{\partial\overline{\psi}}{\partial\overline{x}}\frac{\partial^2\overline{\psi}}{\partial\overline{y}^2}-\frac{\partial\overline{u}_e}{\partial\overline{t}}-\overline{u}_e\frac{\partial\overline{u}_e}{\partial\overline{x}}-\frac{\partial}{\partial\overline{y}}\left(\left|\frac{\partial^2\overline{\psi}}{\partial\overline{y}^2}\right|^{n-1}\frac{\partial^2\overline{\psi}}{\partial\overline{y}^2}\right)+\frac{\sigma_e B_0^2}{\rho}\frac{\partial\overline{\psi}}{\partial\overline{y}} \\ &=\mathscr{H}_1(a_1,a_2)\left[\frac{\partial^2\psi}{\partial t\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}-\frac{\partial u_e}{\partial t}-u_e\frac{\partial u_e}{\partial x}-\frac{\partial}{\partial y}\left(\left|\frac{\partial^2\psi}{\partial y^2}\right|^{n-1}\frac{\partial^2\psi}{\partial y^2}\right)+\frac{\sigma_e B_0^2}{\rho}\frac{\partial\psi}{\partial y}\right] \end{aligned} \tag{9}$$

Using values from equation (8) and applying chain rule for transforming the above derivatives, we get

$$\left(\frac{\square^{\psi}}{\square^{t}\square^{y}}\right)\frac{\partial^2\psi}{\partial t\partial y}+\left(\frac{\left(\square^{\psi}\right)^2}{\square^{t}\left(\square^{y}\right)^2}\right)\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\left(\frac{\left(\square^{\psi}\right)^2}{\square^{x}\left(\square^{y}\right)^2}\right)\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}-\left(\frac{\square^{u_e}}{\square^{t}}\right)\frac{\partial u_e}{\partial t}-\left(\square^{u_e}u_e+\square^{u_e}\right)\left(\frac{\square^{u_e}}{\square^{x}}\right)\frac{\partial u_e}{\partial x}$$
$$-\left(\frac{\left(\square^{\psi}\right)^n}{\left(\square^{y}\right)^2}\right)\left\{n\left(\frac{\partial^2\psi}{\partial y^2}\right)^{n-1}\frac{\partial^3\psi}{\partial y^3}\right\}+M_1+\frac{\sigma_e B_0^2}{\rho}\left(\frac{\square^{\psi}}{\square^{y}}\right)\frac{\partial\psi}{\partial y}$$
$$=\mathcal{H}_1(a_1,a_2)\left[\frac{\partial^2\psi}{\partial t\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}-\frac{\partial u_e}{\partial t}-u_e\frac{\partial u_e}{\partial x}-\frac{\partial}{\partial y}\left(\left|\frac{\partial^2\psi}{\partial y^2}\right|^{n-1}\frac{\partial^2\psi}{\partial y^2}\right)+\frac{\sigma_e B_0^2}{\rho}\frac{\partial\psi}{\partial y}\right] \quad (10)$$

Where $M_1=\frac{\square^{u_e}\square^{u_e}}{\square^{x}}\frac{\partial u_e}{\partial x}$ and for invariance of (10), $M_1=0$ invariance of above equations along with the boundary condition gives:

$$\frac{\square^{\psi}}{\square^{t}\square^{y}}=\frac{\left(\square^{\psi}\right)^2}{\square^{x}\left(\square^{y}\right)^2}=\frac{\square^{u_e}}{\square^{t}}=\frac{\left(\square^{u_e}\right)^2}{\square^{x}}=\frac{\left(\square^{\psi}\right)^n}{\left(\square^{y}\right)^{2n+1}}=\frac{\square^{\psi}}{\square^{y}}=\mathcal{H}_1(a_1,a_2) \quad (11)$$

Also, the boundary conditions in (7) are transformed invariantly, whenever

$$\frac{\square^{\psi}}{\square^{x}}=\square^{u_e},\ \square^{y}=\square^{u_e}=0 \quad (12)$$

Solving these we obtain

$$\square^{\psi}=(\square^{y})^4,\ \square^{x}=(\square^{y})^3,\ \square^{t}=1,\ \square^{u_e}=(\square^{y})^3,\ \square^{y}=\square^{u_e}=0 \quad (13)$$

Here, we get the two-parameter group G which transforms invariantly the equations (6) with the boundary conditions (7). The group G is of the form:

$$G:\begin{cases}\overline{x}=(\square^{y})^3x+\square^{x}\\ \overline{y}=(\square^{y})y\\ \overline{t}=t+\square^{t}\\ \overline{\psi}=(\square^{y})^4\psi+\square^{\psi}\\ \overline{u_e}=(\square^{y})^3u_e\end{cases} \quad (14)$$

3.2 The complete set of absolute invariants

Now, we will obtain a complete set of absolute invariants by transforming the problem into an ordinary differential equation in a similarity variable via deductive group theoretic method.

If $\eta=\eta(x,y,t)$ is the absolute invariant of the independent variables then, the absolute invariants for the dependent variable ψ, u_e are given by

$$g_j(x,y,t;\psi,u_e)=F_j(\eta),\qquad j=1,2,3 \quad (15)$$

The function $g_j(x,y,t;\psi,u_e)$ is an absolute invariant of a two-parameter group if it satisfies the following two first-order linear differential equations:

$$\left.\begin{aligned}&\sum_i(\delta_iS_i+\delta_{i+1})\frac{\partial g}{\partial S_i}=0,\\ &\sum_i(\sigma_iS_i+\sigma_{i+1})\frac{\partial g}{\partial S_i}=0,\end{aligned}\right\}\quad i=1,3,\ldots,9\ and\ S_i=x,y,t,\psi,u_e \quad (16)$$

where $\delta_1 = \left.\dfrac{\partial \square^x}{\partial a_1}\right|_{(a_1{}^0, a_2{}^0)}, \delta_2 = \left.\dfrac{\partial \square^x}{\partial a_1}\right|_{(a_1{}^0, a_2{}^0)}, \sigma_1 = \left.\dfrac{\partial \square^x}{\partial a_2}\right|_{(a_1{}^0, a_2{}^0)}, \sigma_2 = \left.\dfrac{\partial \square^x}{\partial a_2}\right|_{(a_1{}^0, a_2{}^0)}$ *etc.* (17)

and $'(a_1{}^0, a_2{}^0)'$ denotes the value of $'(a_1, a_2)'$ which is the identity element of the group G .

3.2.1 The absolute invariants of independent variables

The absolute invariant $\eta(x, y, t)$ of the independent variables (x, y, t) is obtained using (16) and

(17) as follow:

$$\left.\begin{array}{l} (\delta_1 x + \delta_2)\dfrac{\partial \eta}{\partial x} + (\delta_3 y)\dfrac{\partial \eta}{\partial y} + (\delta_5 t + \delta_6)\dfrac{\partial \eta}{\partial t} = 0 \\ (\sigma_1 x + \sigma_2)\dfrac{\partial \eta}{\partial x} + (\sigma_3 y)\dfrac{\partial \eta}{\partial y} + (\sigma_5 t + \sigma_6)\dfrac{\partial \eta}{\partial t} = 0 \end{array}\right\} \tag{18}$$

Because $\delta_4 = \sigma_4 = 0 \ as \ \square^y = 0$.

By definition, for each of two-parameter group G in the class G there is one and only one functionally independent solution of (18), i.e. the rank of the coefficient matrix for $\begin{bmatrix} \dfrac{\partial \eta}{\partial x} & \dfrac{\partial \eta}{\partial y} & \dfrac{\partial \eta}{\partial t} \end{bmatrix}$ is two, (the matrix has rank two whenever at least one of its two-by-two sub-matrices has a non-vanishing determinant)(Darji & Timol, 2014). According to this at least one term out of:

$\lambda_{13}x + \lambda_{23}, \ \lambda_{35}t + \lambda_{36}, \ \lambda_{15}xt + \lambda_{16}x + \lambda_{25}t + \lambda_{26}$

is non zero, where $\lambda_{ij} = \delta_i \sigma_j - \delta_j \sigma_i$.

Using the definitions of δ 's and σ's from equation (17), we have $\lambda_{13} = \lambda_{35} = \lambda_{15} = \lambda_{25} = 0$.

By eliminating $\dfrac{\partial \eta}{\partial y}, \dfrac{\partial \eta}{\partial x}$ from equation (18), we get

$$\left.\begin{array}{l} \lambda_{32}\dfrac{\partial \eta}{\partial x} + \lambda_{36}\dfrac{\partial \eta}{\partial t} = 0 \\ \lambda_{23} y \dfrac{\partial \eta}{\partial y} - (\lambda_{16} x + \lambda_{26})\dfrac{\partial \eta}{\partial t} = 0 \end{array}\right\} \tag{19}$$

Thus, by the basic theorem of Morgan there exists a unique solution of the above system of equations. This leads to the following cases.

Case-I $\lambda_{23} = 0, \ \lambda_{36} \neq 0, \ \lambda_{16}x + \lambda_{26} \neq 0$

In this case the equation (19) becomes $\dfrac{\partial \eta}{\partial t} = 0$.

This case requires the steady state which is not possible for unsteady flow.

Case-II $\lambda_{23} \neq 0, \ \lambda_{36} \neq 0, \ \lambda_{16}x + \lambda_{26} = 0$

In this case the equation (19) becomes

$$\left.\begin{aligned}&\bar{\lambda}_{32}\frac{\partial\eta}{\partial x}+\bar{\lambda}_{36}\frac{\partial\eta}{\partial t}=0\\&\bar{\lambda}_{32}y\frac{\partial\eta}{\partial y}=0\end{aligned}\right\}\tag{20}$$

The second equation of (20) gives $\frac{\partial\eta}{\partial y}=0$. i.e., η is independent of y which is not possible because of the boundary conditions.

Case-III $\bar{\lambda}_{23}\neq0,\ \bar{\lambda}_{36}=0,\ \bar{\lambda}_{16}x+\bar{\lambda}_{26}\neq0$

In this case the equation (19) becomes

$$\left.\begin{aligned}&\frac{\partial\eta}{\partial x}=0\\&\bar{\lambda}_{23}y\frac{\partial\eta}{\partial y}-(\bar{\lambda}_{16}x+\bar{\lambda}_{26})\frac{\partial\eta}{\partial t}=0\end{aligned}\right\}\tag{21}$$

Here, the solution of above system is independent of x, thus $\eta=\eta(y,t)$.

Since η is independent of x we must have $\bar{\lambda}_{16}=0$,this gives

$$\bar{\lambda}_{23}y\frac{\partial\eta}{\partial y}-\bar{\lambda}_{26}\frac{\partial\eta}{\partial t}=0$$

From standard technique for linear partial differential equation, we obtain

$$\frac{dy}{\bar{\lambda}_{23}y}=-\frac{dt}{\bar{\lambda}_{26}}$$

On solving above equation, we get

$$\eta=\eta(y,t)=ye^{\frac{\bar{\lambda}_{23}}{\bar{\lambda}_{26}}t}=y\Pi(t),\text{ where }\Pi(t)=e^{At},\ A=\frac{\bar{\lambda}_{23}}{\bar{\lambda}_{26}}\tag{22}$$

3.2.2 The absolute invariants of dependent variables

Now we obtain the absolute invariant of dependent variables ψ,u_e.

If a function $g_1(x,t,\psi)$ satisfies two linear partial differential equations, then it is an absolute invariant of $\psi(x,y,t)$for two parameter group G, given by

$$\left.\begin{aligned}&(\delta_1x+\delta_2)\frac{\partial g_1}{\partial x}+(\delta_5t+\delta_6)\frac{\partial g_1}{\partial t}+(\delta_7\psi+\delta_8)\frac{\partial g_1}{\partial\psi}=0\\&(\sigma_1x+\sigma_2)\frac{\partial g_1}{\partial x}+(\sigma_5t+\sigma_6)\frac{\partial g_1}{\partial t}+(\sigma_7\psi+\sigma_8)\frac{\partial g_1}{\partial\psi}=0\end{aligned}\right\}\tag{23}$$

The solution of these equations gives

$$g_1(x,t,\psi)=\Phi_1\left\{\frac{\psi}{\Gamma(x,t)}\right\}=F(\eta)\tag{24}$$

In similar way we can obtain

$$g_2(x,t,u_e)=\Phi_2\left\{\frac{u_e}{\omega(x,t)}\right\}=E(\eta)\tag{25}$$

Where $\Gamma(x,t),\ \omega(x,t)$ are functions to be determined and without loss of generality we consider the arbitrary functions Φ's in (24)-(25) to be the identity functions, whenever

$$\psi(x,y,t)=\Gamma(x,t)F(\eta) \tag{26}$$

$$u_e(x,t)=\omega(x,t)E(\eta) \tag{27}$$

Since $\Gamma(x,t),\ \omega(x,t)$ are independent of y whereas η depends on y. Thus, in equation (27) we can say that $E(\eta)$ must be constant say E_0, that is

$$u_e(x,t)=E_0\omega(x,t) \tag{28}$$

Here, we assuming $u_e(x,t)=u_e(x)=ax$, where a is some arbitrary constant. (29)

The forms of $\Gamma(x,t),\ \omega(x,t)$ are those for which equation (6) will be reduced to ordinary differential equations.

3.3 The reduction to an ordinary differential equation

Substituting the values in equation (6) from equations (22), (26), and (29) we get

$$\begin{aligned}&\left(-n\Gamma^{n}\Pi^{2n+1}\right)\left(F''\right)^{n-1}F'''+\eta\Gamma\frac{\partial\Pi}{\partial t}F''+\Pi\frac{\partial\Gamma}{\partial t}F'+\Gamma\frac{\partial\Pi}{\partial t}F'+\Pi\Gamma^{2}\frac{\partial\Pi}{\partial x}\left(F'\right)^{2}+\Gamma\Pi^{2}\frac{\partial\Gamma}{\partial x}\left(F'\right)^{2}\\&-\Gamma\Pi^{2}\frac{\partial\Gamma}{\partial x}FF''-a^{2}x+\frac{\sigma_e B_0^2}{\rho}\Gamma\Pi F'=0\end{aligned} \tag{30}$$

Where prime refers to the derivative with respect to η.

By dividing the above equation (30) with leading coefficient, we get

$$\begin{aligned}&\left(F''\right)^{n-1}F'''-\frac{\left(\frac{\partial\Pi}{\partial t}\right)}{n\Gamma^{n-1}\Pi^{2n+1}}\eta F''-\frac{\left(\frac{\partial\Gamma}{\partial t}\right)}{n\Gamma^{n}\Pi^{2n}}F'-\frac{\left(\frac{\partial\Pi}{\partial t}\right)}{n\Gamma^{n-1}\Pi^{2n+1}}F'-\frac{\left(\frac{\partial\Pi}{\partial x}\right)}{n\Gamma^{n-2}\Pi^{2n}}\left(F'\right)^{2}-\frac{\left(\frac{\partial\Gamma}{\partial x}\right)}{n\Gamma^{n-1}\Pi^{2n-1}}\left(\left(F'\right)^{2}-FF''\right)+\frac{a^{2}x}{n\Gamma^{n}\Pi^{2n+1}}\\&-\frac{\sigma_e B_0^2}{\rho}\frac{1}{n\Gamma^{n-1}\Pi^{2n}}F'=0\end{aligned} \tag{31}$$

To reduce equation (31) into an ordinary differential equation of single variable η, then the coefficients of the function $F(\eta)$ and their derivatives must be either constants or functions of η only. These coefficients are considered as:

$$\begin{aligned}&D_1=\frac{\left(\frac{\partial\Pi}{\partial t}\right)}{n\Gamma^{n-1}\Pi^{2n+1}},\quad D_2=\frac{\left(\frac{\partial\Gamma}{\partial t}\right)}{n\Gamma^{n}\Pi^{2n}},\quad D_3=\frac{\left(\frac{\partial\Pi}{\partial x}\right)}{n\Gamma^{n-2}\Pi^{2n}},\quad D_4=\frac{\left(\frac{\partial\Gamma}{\partial x}\right)}{n\Gamma^{n-1}\Pi^{2n-1}},\quad D_5=\frac{a^{2}x}{n\Gamma^{n}\Pi^{2n+1}},\\&D_6=\frac{\sigma_e B_0^2}{\rho}\frac{1}{n\Gamma^{n-1}\Pi^{2n}}\end{aligned} \tag{32}$$

where the D's are constants to be determined

Thus, by using equation (32) in (31), we obtain equation as

$$\left(F''\right)^{n-1}F'''+(D_4F-D_1\eta)F''-(D_3+D_4)\left(F'\right)^{2}-(D_6+D_1+D_2)F'+D_5=0 \tag{33}$$

with the boundary conditions

$$\begin{aligned}&F(0)=0=F'(0), && \eta=0\\&F'(\infty)=1, && \eta=\infty\end{aligned} \tag{34}$$

3.4 To find the unknown functions

Case-III In this case $\eta = y\Pi(t) = ye^{At}$

Which gives $\frac{\partial \Pi}{\partial x} = 0$ and $\frac{\partial \Pi}{\partial t} = Ae^{At} = A\Pi$

Substituting these values in (32) and solving them we can obtain

$\Gamma(x,t) = Ke^{At} = K\Pi$, K is some constant. (35)

$D_1 = D_2;\ \ D_3 = D_4 = 0$

(36)

Substituting the obtained constants in equation (33), yields

$$(F'')^{n-1}F''' - D_1\eta F'' - (D_6 + 2D_1)F' + D_5 = 0 \tag{37}$$

Together with the boundary conditions

$$\begin{aligned} &F(0) = 0 = F'(0), && \eta = 0 \\ &F'(\infty) = 1, && \eta = \infty \end{aligned} \tag{38}$$

The stream function is given by

$$\psi(x,y,t) = Ke^{At}F(\eta) \text{ where } \eta = ye^{At},\ A = \frac{\lambda_{23}}{\lambda_{26}} \tag{39}$$

4. Numerical solution

The system of reduced non-linear ordinary differential equation (37) along with the boundary conditions (38) is solved numerically using bvp4c solver in MATLAB. The step size 0.002 is used to obtain the solution for the interval from 0 to 1.

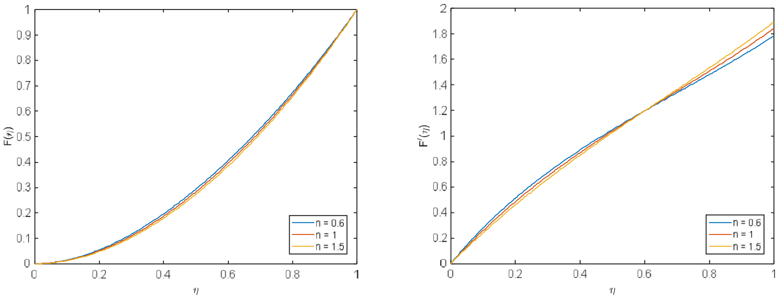
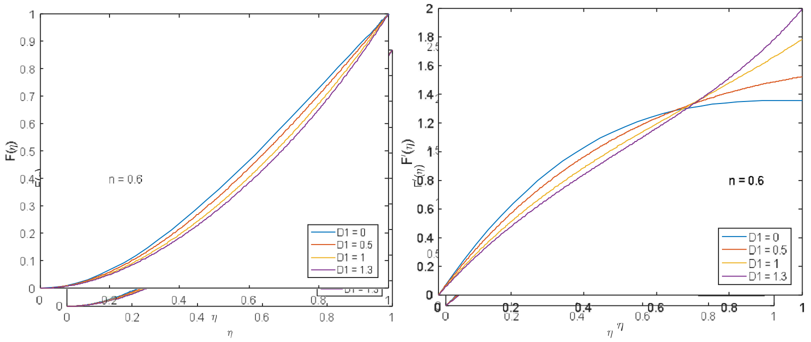


Figure 1. Impact of different values of fluid index n (Power Law Fluid) on $F(\eta)$ and velocity profile $F'(\eta)$ for $D_1 = 1,\ D_5 = 4,\ D_6 = 0.2$



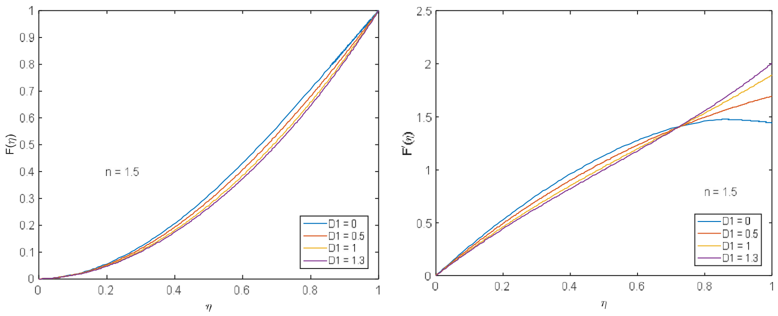


Figure 2. Impact of different values of parameter D_1 on $F(\eta)$ and velocity profile $F'(\eta)$ for $D_5 = 4$, $D_6 = 0.2$ and $n = 0.6, 1, 1.5$

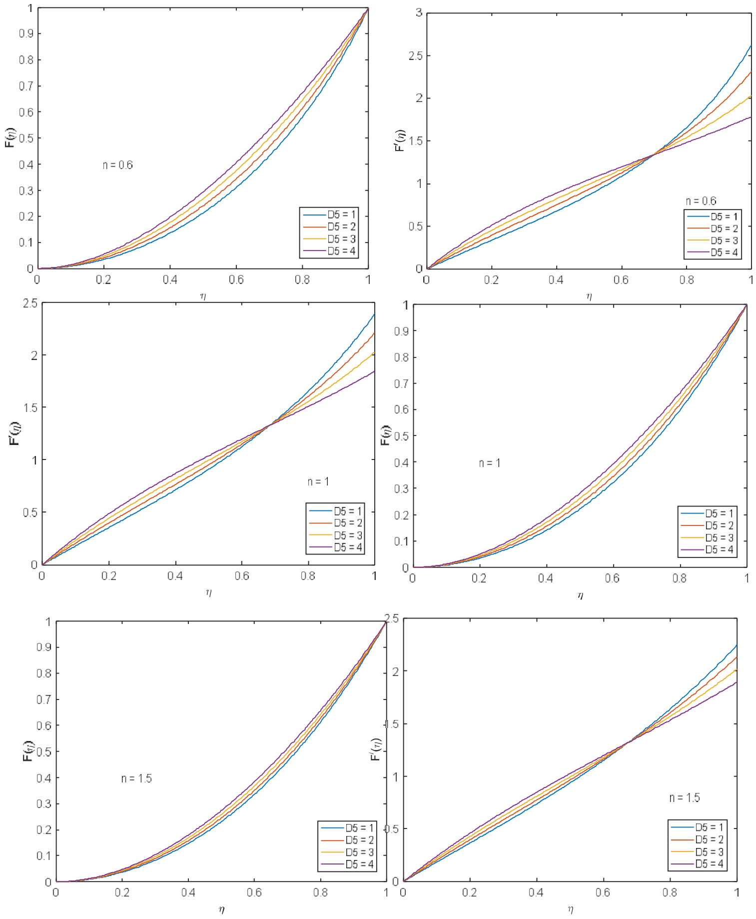


Figure 3. Impact of different values of parameter D_5 on $F(\eta)$ and velocity profile $F'(\eta)$ for $D_1 = 1$, $D_6 = 0.2$ and $n = 0.6, 1, 1.5$

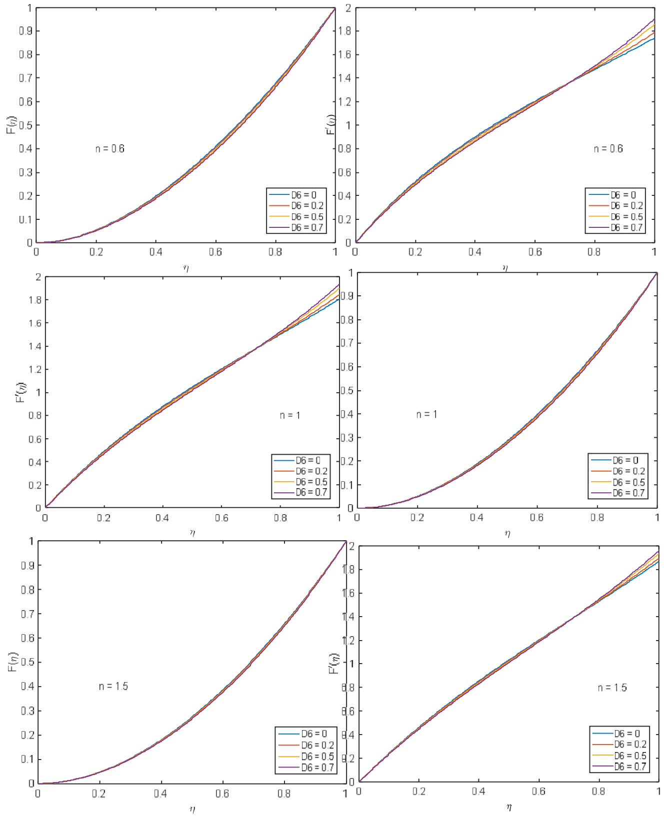


Figure 4. Impact of different values of parameter D_6 on $F(\eta)$ and velocity profile $F'(\eta)$ for $D_1 = 1$, $D_5 = 4$ and $n = 0.6, 1, 1.5$

5. Results and Discussion

In the present section, we investigate the effects of various parameters such as D_1, D_5, D_6 and fluid index n (Power law fluid) on $F(\eta)$ and velocity profile $F'(\eta)$ and presented graphically in Figures 1 to 4. The graph of velocity versus η interprets that as value of η tends to infinity, velocity starts at the wall with a value zero and approaches to the outer flow velocity. The graph curve of $F(\eta)$ and velocity profile $F'(\eta)$ depends on Power-law fluid index, transverse magnetic field parameter, and unsteadiness flow parameter.

Effects on $F(\eta)$ and velocity $F'(\eta)$:

Figures 1 presents the graph of $F(\eta)$ and velocity profile $F'(\eta)$ for both Newtonian (n = 1) and Non-Newtonian (for n < 1 pseudoplastic and for n > 1 dilatant) fluids by substituting different values of fluid index n (Power law fluid). Here, parameters D_1, D_5, and D_6 are kept fixed, and the effects on the $F(\eta)$ and the velocity profile $F'(\eta)$ are shown for Shear thinning (n < 1), Newtonian (n = 1), and Shear thickening (n > 1) of the Power Law fluid. From the figure, it is clear that with an increase in the value of fluid index n (Power law fluid) there is an increase in the $F(\eta)$ and the velocity profile $F'(\eta)$, which results in the increase of thickness of fluid near the surface of the plate. Hence, with the increase in fluid index n, it generates resistance in the fluid.

In Figure 2, parameters D_5 and D_6 are kept fixed; the fluid index n is considered for three cases (pseudoplastic, Newtonian and dilatant), and the effect of parameter D_1 on $F(\eta)$ and the velocity profile $F'(\eta)$ is shown. Here, $F(\eta)$ and the velocity profile $F'(\eta)$ increase with the increase in the parameter D_1.

In Figure 3, parameters D_1, D_6 are kept fixed; the fluid index n is considered for three cases (pseudoplastic, Newtonian and dilatant) and the effect of parameter D_5 on $F(\eta)$ and the velocity profile $F'(\eta)$ is shown. Here, $F(\eta)$ and the velocity profile $F'(\eta)$ increase with the increase in the parameter D_5 .

In Figure 4, parameters D_1, D_5 are kept fixed; the fluid index n is considered for three cases (pseudoplastic, Newtonian and dilatant) and the effect of parameter D_6 on $F(\eta)$ and the velocity profile $F'(\eta)$ is shown. Here, $F(\eta)$ and the velocity profile $F'(\eta)$ increase with the increase in the parameter D_6 .

6. Conclusion

In the present paper, the governing non-linear partial differential boundary layer equation of non-Newtonian fluids for stagnation point flow of power law fluid over a porous flat plate in the presence of uniform transverse magnetic field is investigated on $F(\eta)$ and the velocity profile $F'(\eta)$ using similarity solutions. Deductive group-theoretic method is applied for the similarity transformation of non-Newtonian power law fluid over a porous medium.

The effects of Power law fluid index and various parameters on a stagnation point flow over a porous medium is investigated numerically for both Newtonian and Non-Newtonian power law fluids by using MATLAB bvp4c. The conclusion of the numerical results is summarized as:

- Here graph of $F(\eta)$ and the velocity $F'(\eta)$ versus fluid index n, shows the behaviour of the fluid for shear thinning ($n < 1$), Newtonian ($n = 1$), and shear thickening ($n > 1$).
- The graph of different parameters D_1, D_5 and D_6 shows the behaviour of $F(\eta)$ and the velocity profile $F'(\eta)$ for Newtonian ($n = 1$) and non-Newtonian (for $n = 0.6$ pseudoplastic and for $n = 1.5$ dilatant).
- On an electrically conducting fluid, when a transverse magnetic field is applied, it creates resistance in the fluid, which results in creating the resistance in velocity due to which there is delayed increase in velocity.

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